MOTION OF A SOLID PARTICLE IN A ROTATING POTENTIAL FLOW

N. I. Zverev and S. G. Ushakov

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It is shown that the process of stabilizing the motion of a particle in a rotating potential flow takes the form of aperiodic damped oscillations.

A possible motion in centrifugal equipment is equilibrium steady-state circular motion under the action of equal and opposite centrifugal and drag forces. The radius r_p of the circle is determined by the dimensions of the particle and the flow, the flow velocity, and the physical constants of the gas and the particle. It is often assumed [1, 2] that if for a given particle $r_{in} < r_p < r_{out}$ (r_{in} and r_{out} are the radii of the central outlet and outer wall of the equipment, respectively), the particle revolves indefinitely around this equilibrium circle and will enter the fine or coarse fraction only as a result of various random influences: collisions with other particles, turbulent fluctuations of the flow, etc.

We attempted a more complete investigation of the motion of a spherical particle in a plane rotating flow with central outlet and vertical axis.

The differential equations of motion of a dust particle in polar coordinates r, φ have the form

$$j_r = \frac{F_r}{m} = \frac{dw_r}{dt} - \frac{w_\varphi^2}{r}, \qquad (1)$$

$$j_\varphi = \frac{F_\varphi}{m} = \frac{dw_\varphi}{dt} + \frac{w_r w_\varphi}{r},$$

$$F = c\frac{\pi\delta^2}{4}\frac{\rho_1 u^2}{2} = c\,\mathrm{Re}\frac{\pi\eta\delta u}{8} = \psi\frac{\pi\eta\delta u}{8}. \qquad (2)$$

Substituting for F_r and F_φ in (1) and (2)

$$\frac{dw_r}{dt} = \frac{w_\varphi^2}{r} + \psi\frac{\pi\eta\delta}{8m}(v_r - w_r), \qquad (3)$$

$$\frac{dw_\varphi}{dt} = -\frac{w_r w_\varphi}{r} + \psi\frac{\pi\eta\delta}{8m}(v_\varphi - w_r). \qquad (4)$$

To these equations we must add the two kinematic equations

$$\frac{dr}{dt} = w_r, \qquad (5)$$

$$\frac{d\varphi}{dt} = \frac{w_\varphi}{r}. \qquad (6)$$

Dividing (3), (4), and (5) by (6) and expressing r, v, and w in terms of the characteristic quantities r_0 and v_0, we obtain the following system of dimensionless differential equations of motion of a dust particle:

$$\frac{d\rho}{d\varphi} = \rho\frac{W_r}{W_\varphi}, \qquad (7)$$

$$\frac{dW_r}{d\varphi} = W_\varphi + \frac{\psi\rho}{\mathrm{St}\,W_\varphi}(V_r - W_r), \qquad (8)$$

$$\frac{dW_\varphi}{d\varphi} = -W_r + \frac{\psi\rho}{\mathrm{St}\,W_\varphi}(V_\varphi - W_r). \qquad (9)$$

Consider the case of potential rotation

$$V_\varphi = \frac{1}{\rho}, \quad V_r = -\frac{\mathrm{tg}\,\alpha}{\rho},$$

where $\mathrm{tg}\,\alpha = V_r/V_\varphi$.

If for steady-state rotation of the particle we set

$$\frac{dW_r}{d\varphi} = 0, \quad W_r = 0, \quad W_\varphi = V_\varphi, \quad \rho = \rho_p = \frac{r_p}{r_0},$$

in (8), we obtain

$$U = \mathrm{tg}\,\alpha\sqrt[3]{\frac{R^2}{\mathrm{St}}} = \sqrt[3]{\frac{\mathrm{Re}^2}{\psi}}$$

for the dimensionless critical velocity in the centrifugal force field;

$$D = \frac{\sqrt[3]{\mathrm{St}\,R}}{\rho_p} = \sqrt[3]{\psi\,\mathrm{Re}}$$

is the dimensionless particle diameter.

After evaluating U we find Re, ψ, D from tables of the function $\psi = f(\mathrm{Re})$ and then ρ_p.

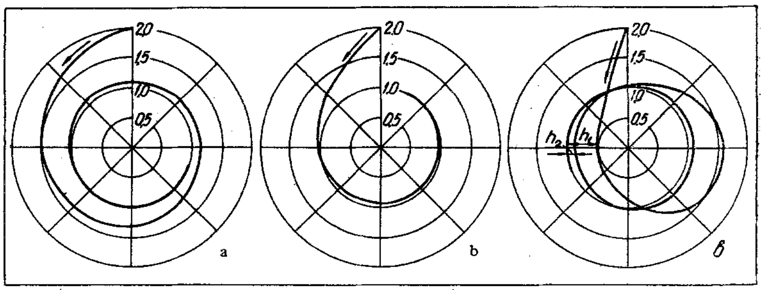

Fig. 1. Particle trajectories in a rotating potential flow (St = 32, R = 64): a) tg α = 0.3; b) 1.0; c) 3.0.

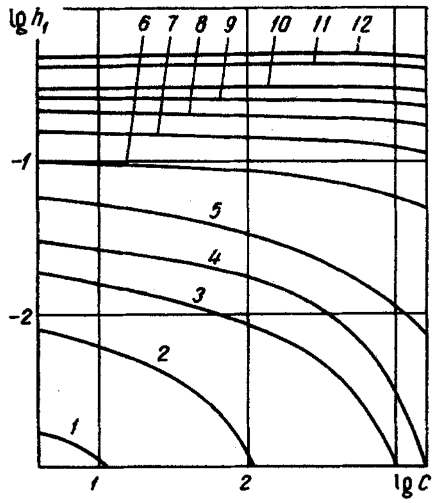

Fig. 2. Maximum particle overshoot as a function of the characteristic parameters: 1) tg α = 0.6; 2) 0.7; 3) 0.8; 4) 0.9; 5) 1.0; 6) 1.2; 7) 1.4; 8) 1.6; 9) 1.8; 10) 2.0; 11) 2.6; 12) 3.0.

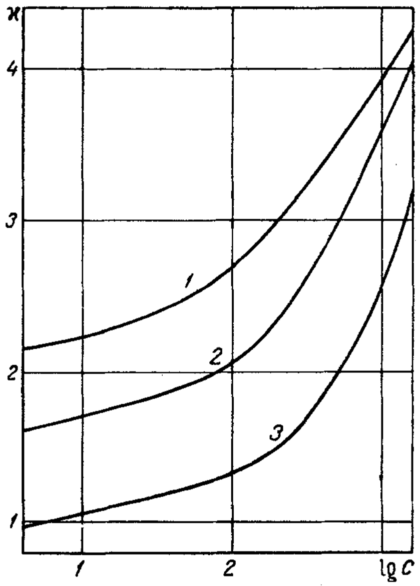

Fig. 3. Logarithmic decrement as a function of the characteristic parameters: 1) tg α = 1.6; 2) 2.0; 3) 3.0.

We assume that the particle is introduced into the flow at the point $\varphi_0 = 0$, $\rho_0 = 2\rho_p$ with a velocity equal to the flow velocity at that point.

System (7)–(9) was integrated numerically by a modified Runge-Kutta method. The values of ψ are experimental and were taken from $\psi = f(\mathrm{Re})$ tables. The Re number was calculated from the equation

$$\mathrm{Re} = R\sqrt{(V_r - W_r)^2 + (V_\varphi - W_\varphi)^2}.$$

The calculations were carried out on a "Ural-2" computer. The computation error was $\varepsilon \leq 0.00001$. A total of 260 variants were calculated for various values of the parameters

$$\mathrm{St} = 10^{-6} - 4\cdot 10^{6}, \quad R = 10^{-3} - 1.3\cdot 10^{5},$$

$$\mathrm{tg}\,\alpha = 0.3 - 3.0.$$

The typical particle trajectories presented in Fig. 1a, b, c correspond to identical values of the parameters St = 32 and R = 64, but different values of tg α (tg α = 0.3, 1.0, 3.0). The radius is expressed in fractions of ρ_p.

From an inspection of the trajectories we draw the following conclusions:

1) the particle approaches the steady state ($\rho = \rho_p$ = const) by a process of aperiodic, rapidly damped oscillations, with inertial overshoots on both sides of the equilibrium trajectory, and

2) the overshoots increase sharply as the degree of twist decreases (tg α increases).

The maximum overshoot h_1 (Fig. 1c) as a function of St and R is represented by a family of curves. If we substitute for St and R the derived parameters $C = R^2/\mathrm{St} = \mathrm{Re}_0(\rho_1/\rho_2)$ (where $\mathrm{Re}_0 = (3/4)(v_{\varphi_0} r_0 \rho_1/\eta)$ is the Reynolds number for the flow) and $\Delta = \mathrm{St}/R = (4/3)(\delta\rho_2/r_0\rho_1)$, then for each tg α the values of h_1 fit a single curve (Fig. 2), i.e., in this case the process is self-similar with respect to the parameter Δ.

As C increases, the maximum inertial overshoot decreases and the logarithmic decrement of the particle oscillations $\varkappa = \ln(h_1/h_2)$ grows (Fig. 1c, 3), i.e., the steady state of motion along the equilibrium trajectory is more rapidly approached.

NOTATION

δ and ρ_2 are the diameter (m) and the density (kg/m^3) of a dust particle; η and ρ_1 are the dynamic viscosity (N · sec/m^2) and density (kg/m^3) of the gas; m is the mass of a dust particle (kg); w and v are the velocities of the dust particle and the gas (m/sec), respectively; u is the gas velocity relative to a dust particle (m/sec); j is the acceleration of the dust particle (m/sec^2); F is the drag force (N); t is the time (sec); c and ψ are the quadratic and linear drag coefficients; $\mathrm{Re} = u\delta\rho_1/\eta$ is the Reynolds number for the dust particle; $\mathrm{St} = 4\delta^2 v_{\varphi_0}\rho_2/3\eta r_0$ is the Stokes number; $R = \delta v_{\varphi_0}\rho_1/\eta$ is a dimensionless number; $V = v/v_0$, $W = w/v_0$ are the dimensionless gas and dust particle velocities; and $\rho = r/r_0$ is the dimensionless radius.

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Dzerzhinskii All-Union Heat Engineering Institute, Eastern Branch, Chelyabinsk